Hazardous facility location models on networks

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1 Introduction

Ever since the most ancient civilizations, human beings have always sought for the best place to live. Nice weather, pleasant environmental conditions, wealth of food and water, and safeness against external harms, are some of the most important issues for choosing the best spot where a new settlement should be established.

Most of the papers most regarding location problems address the siting of facilities such as emergency services (police/fire stations), educational centers, medical facilities, etc., that are considered desirable by the surrounding population. However, there are some other facilities such as garbage dump sites, landfills, chemical plants, nuclear reactors, military installations and polluting (noise/gas) plants that turn out to be undesirable (repulsive) for the surrounding population, that avoids them and tries to stay away from them. In this sense, Erkut and Neuman (1989) distinguish between *noxious* (hazardous) and *obnoxious* (nuisance) facilities, although both can be simply regarded as *undesirable*.

Despite these undesirable facilities being necessary in general to the community, for instance, garbage dump sites, gas stations, electrical plants, etc., the location of such facilities might cause a certain disagreement among the population. Such a disagreement may result in a true opposition of people to the installation of undesirable facilities in their neighborhood. Moreover, in the last decade, a new nomenclature has been developed to define these oppositions: NIMBY (*Not In My Back Yard*), NIMNBY (*Not In My Neighbor's Back Yard*), NIABY (*Not In Anyone's Back Yard*), NIMTOO/NIMTOF (*Not In My Term of Office*), NOPE (*Not On Planet Earth*), LULU (*Locally Unwanted Land Use*), BANANA (*Build Absolutely Nothing Anywhere Near Anyone*).

Taking these concerns into account, and due to the great concern on environmental issues that has arisen in the last decades, this chapter aims to analyze some undesirable facility location models, preferably on networks.

On the other hand, network location models have usually dealt with single criterion problems, that is, concerning one weight per node and/or one length per edge. However, to properly model many real problems the decision maker requires placing parameters on both the nodes such as demand, importance, number of customers, etc., and on the edges such as length, time, travel cost, etc. Many researchers, in several excellent reviews and books, for instance, ReVelle, Cohon and Shobrys (1981a,b), Ross and Soland (1980), Krarup and Pruzan (1990), Current, Min and Schilling (1990), Daskin (1995), have deeply emphasized the importance of dealing with multiple objectives in Location Analysis.

Furthermore, many authors have deeply argued in the literature that a lot of multicriteria/multiobjective location problems have remained unresearched even though this topic has become quite relevant in the last three decades. In this sense, Erkut and Neuman (1989) emphasized on the need for multiobjective approaches to the siting of undesirable facilities when they stated that (p. 289): *"Current models can be used to generate a small number of candidate sites, but the final selection of a site is a complex problem and should be approached using multiobjective decision making tools"*. Daskin (1995) and Zhang (1996) also pointed out not only the need to include multiple criteria in undesirable facility location problems, but also the fact that poor attention has been paid by researchers to these problems and hence, scarce research has been done in this promising field. Therefore, section 6 of this chapter mainly focuses on network location models concerning multiple criteria, in terms of considering several node weights and several edge lengths.

In the remaining paragraphs we summarize the contents of the chapter. In Section 2 we justify the importance of Network Location Models within the field of Location Theory. Section 3 allows the reader to get acquainted with the definition and classical literature in Location Theory. In this respect, more than 80 references are reviewed, from surveys and books in general location problems, to more specialized papers on multicriteria undesirable location models on networks. Section 4 presents the basic definitions and notation used throughout the chapter for the standard networks and also for networks with multiple parameters on both nodes and edges. The problem of locating an undesirable facility on a network is addressed in Section 5. Section 6 is devoted to the location of undesirable facilities on multicriteria networks. In section 7 we summarize the conclusions and describe some open problems that may be researched with regards to the location/transportation of hazardous materials. Finally, the last section lists all the bibliography referenced.

2 Network location models within Location Theory

In a very wide sense, location problems deal with finding the right site where one or more new facilities (services) should be placed, in order to optimize (minimize or maximize) some specified criteria, which are usually related to the distance (performance measure) from the facilities to the demand points (customers).

The mathematical field that formulates location problems, builds up appropriate mathematical models and derives methods for solving them is called ***Location Theory*** or ***Location Analysis***. Being a branch of the *Operational Research* framework, this subject provides decision-makers with qualitative tools to find good solutions for realistic location decision problems. Besides, modern Location Analysis has drawn the interest of practitioners such as economists, geographers, regional planners and architect researchers, as well as researchers in diverse fields like *Industrial Engineering*, *Management Science* and *Computer Science*.

Regarding location theory classification, location problems mostly fall in one of the following three categories:

- *Continuous location*: locations are allowed to be anywhere in a continuous *d* dimensional space.
- *Discrete location*: a finite number of possible locations on the space are specified in advance. Sometimes it is also called location-allocation.
- *Network location*: special kind of location problems which are modeled on networks or trees.

In this chapter we focus on *Network Location Problems*. This type of problems can model real location problems on river networks, air transport networks (flight corridors), ocean transport networks (shipping lanes); highways, roads, avenues and street networks; and communication and computer networks. The literature on network location is full of inherent real applications, some of which will be briefly mentioned in the next literature review.

Despite most of these location problems seeming to be close related to the contemporary world, they have been originally proposed centuries ago. This is described in the next section where we present a brief historical background, as well as a comprehensive review of the literature on Network Location Analysis. After this, we introduce a general notation and basic concepts in Location Theory. These concepts are used to describe the models developed in the following sections.

3 Brief historical background and review of the literature

The origin of modern location theory is credited to A. Weber (1909), who incorporated the original problem proposed by Fermat into Location Analysis in his influential essay on the theory of industrial location *"Über den Standort der Industrien"* (Theory of the location of industries), translated later by Friedrich (1929).

Jordan (1869) obtained a characterization of the median set of a tree. With regards to location problems on general networks, we must mention Hakimi (1964), who introduced both the median and the center on weighted networks, and thus, his principal paper set the foundations for the development of forthcoming network location problems.

Literature on Location Analysis is extremely huge and fairly interlaced. One of the first and most extensive compilations is due to Domschke and Drexl (1985), who compiled a bibliography of over 1800 papers. Later on, Drezner (1995) provided more than 1200 references. Trevor Hale (1998) keeps a web page with a list of over 3400 location science, facility location and related references. And this number keeps counting!

Next, we cite some reviews, surveys and books on classical location problems.

3.1 Surveys, reviews and books on location problems

For a reader not quite acquainted with Location Analysis, we now cite some interesting bibliography on classical location problems and models.

A classical and state-of-the-art text on discrete location problems is due to Mirchandani and Francis (1990). Drezner (1995) presented a wide-ranging survey of location analysis. Drezner and Hamacher (2004) covered theory, methodology and selected applications of Location Analysis. Eiselt and Sandblom (2004) present a unified treatment of decision analysis, location theory and scheduling, with topics ranging from multicriteria decision-making to location and layout planning.

Chan (2005) describes procedures to perform site location, land-use planning, location-routing, competitive allocation of products/services and spatial forecasting. Nickel and Puerto (2005) address the flexible location problem called the Ordered Median Problem (OMP), presenting both structural properties and solution approaches of the OMP for continuous, network and discrete location problems.

Some of the latest books on Location Analysis are from Farahani and Hekmatfar (2009), who describe the four main parts (customers, facilities, space and metrics) for each specific location model exemplified by real-world cases; and from Eiselt and Marianov (2011), who compile several contributions written by eminent experts in the field of location analysis, surveying the original seminal papers and providing an up-to-date review of the latest references.

Since the main goal of this chapter is to describe the major achievements on network location regarding hazardous facilities, in the subsequent sections we review, in chronological order, the most outstanding references on location of undesirable facilities on networks considering both one single criterion and several criteria.

3.2 Undesirable facility location problems on networks

There are not many papers devoted to location of undesirable (sometimes called ***obnoxious***) facilities on networks. This subject barely emerged in mid 70s, and gradually drew the interest of researchers due to environmental issues. These types of problems are the opposite of the classical center (*minimax*) and median (*minisum*) problems, and hence, they are usually modeled using the *maximin* and the *maxisum* criteria. Other authors established alternative criteria which are not covered in this chapter. Slater (1975) defined the security center and security centroid of a graph using the criterion that a vertex *u* is "more central" than vertex *v* if there are more vertices closer to *u* than to *v*.

In the same way as Hakimi is the forerunner of Network Location Analysis, Church and Garfinkel (1978) are the precursors of the location of undesirable facilities on networks. They dealt with the problem of locating a point on a network so as to maximize the sum of its weighted distances (*maxisum*) to the nodes, and proposed an algorithm in $O(mn \log n)$ time. The optimal point was called *maxian*. Minieka (1983) characterized the *anticenter* and *antimedian* location models. The former is formulated as a *maxmax* problem, whereas the latter is a directed approach to that of Church and Garfinkel (1978).

Ting (1984) treated the problem of locating a single facility in a tree network considering the maxisum criterion, and provided a solution algorithm with computational effort $O(n)$. Kuby (1987) pointed out that the optimal maximin objective value could be used as a lower bound on the distances between selected facilities. Moon (1989) addressed the problem of finding a point in a tree network whose distance to the closest pendant vertex (incident to a single edge) is maximal. He presented a polynomial time algorithm in $O(n)$ time.

Tamir (1988) demonstrated that for some center and (obnoxious) location problems it is possible to take advantage of dynamic data structures to achieve better complexity bounds. Labbé (1990) dealt with the location of an obnoxious facility on a network using a voting procedure. She also defined the *anti-Condorcet* point as a point such that no other point is farther from a strict majority of users. Tamir (1991) discussed new complexity results for several models dealing with the location of obnoxious or undesirable facilities on graphs such as *p*-maximin and *p*-maxisum problems, which concern the location of *p* facilities under the maximin and maxisum objectives, respectively.

Regarding location and routing of hazardous wastes, Stowers and Palekar (1993) developed a combined model that quantifies the total exposure of the population during transportation as well as long term storage.

Kincaid and Berger (1994) studied the problem of selecting a subset of size *p* of the distance matrix column indices such that the smallest row sum in the resulting $n \times p$ submatrix is as large as possible. Drezner and Wesolowsky (1995) considered the problem of locating a point that should be as far as possible from arcs and nodes of a network. Berman, Drezner and Wesolowsky (1996) approached the location of a new facility on a network so that the total number (weight) of nodes within a prespecified distance is minimized.

Moreno-Pérez and Rodríguez-Martín (1999) addressed the problem of locating an undesirable facility on a network maximizing a convex combination of the average and minimum distance to the population. Since this is the opposite of the cent-dian model, they called it the *anti-cent-dian*. The same problem including distance constraints was previously pointed out by Moon and Chaudhry (1984) as the *anticenter-maxian* model. Colebrook and Sicilia (2006) improved the anti-cent-dian facility location problem on networks, providing an efficient $O(mn)$ time algorithm.

Although Tamir (1988, 2001) already presented a brief $O(mn)$ method for the maximin problem, Melachrinoudis and Zhang (1999) solved the location of a point on a network under the maximin criterion with the same computational effort. Soon after, Berman and Drezner (2000) developed the same problem from a linear programming viewpoint in $O(mn)$ time as well. Colebrook, Gutiérrez, Alonso and Sicilia (2002) presented a different model formulation and improved upper bounds for the location of an undesirable (obnoxious) center on general networks, which diminished the computational time required to get the solution.

Salhi, Welch and Cuninghame-Green (2000) provided an alternative analytical approach to the Voronoi based method for the weighted 1-maximin location problem, which concerns the location of one facility under the maximin criterion. Their enhanced method was relying on two reduction tests and a suitable branch and bound scheme. Zhang, Hodgson and Erkut (2000) developed an algorithm to safely route hazardous materials on network, assessing the potential risks on human population by GIS techniques.

Burkard, Dollani, Lin and Rote (2001) derived algorithms with linear running time in the cases where the network is a path or a star, and improved previous results proposed by Tamir (1988, 1991). In a quite similar approach, Burkard and Dollani (2003) studied the pos/neg 1-center problem on networks, which asks to minimize a linear combination of the maximum weighted distance of the center to the positive and negative weighted vertices respectively. On networks, they provided an $O(mn\log n)$ algorithm, whereas on star graphs the problem can be solved in linear time. They also studied the extensions to the location of p facilities on trees.

López-de-los-Mozos and Mesa (2001) analyzed a new locational equity measure defined as the maximum absolute deviation. They investigated its properties and proposed an algorithm for locating a single facility on a network such that it minimizes this new criterion. Carrizosa and Conde (2002) addressed a p-facility location for semi-desirable facilities whose location was restricted to the edges of a planar network with rectilinear edges.

Cappanera, Gallo and Maffioli (2003) addressed the problem of simultaneously locating obnoxious facilities and routing obnoxious materials between a set of built-up areas and the facilities, defining a discrete combined location-routing model referred to as the Obnoxious Facility Location and Routing model (OFLR).

Berman and Wang (2004) considered, among others, the 1-antimedian and 1-maximin undesirable facility location problems on undirected networks with node weights as independent discrete random variables. Colebrook, Gutiérrez and Sicilia (2005) studied the problem of locating an undesirable facility on a network so as to maximize its total weighted distance to all nodes, giving a new upper bound and a new algorithm in $O(mn)$ time.

Berman and Wang (2006) studied the 1-median and 1-antimedian problems with probabilistic node demands, which are assumed to be independent continuous random variables, whereas Berman and Wang (2007) considered the problem of locating semi-obnoxious facilities provided that some demand points, within a certain distance from an open facility, are expropriated. In Berman and Wang (2008), the problem of locating a semi-obnoxious facility was considered assuming that close demands nodes could be expropriated by the developer.

Erkut and Alp (2007) considered the problem of selecting routes for hazardous material transportation, applying their model to the road network of Ravenna (Italy). Berman, Verter and Kara (2007) presented a novel methodology based on arc-covering to determine the network optimal design so as to maximize the ability to respond to dangerous incidents. Their results assessed the emergency response capability to transport incidents in Quebec and Ontario (Canada).

Recently, Berman and Huang (2008) compared several mathematical formulations to locate undesirable facilities on a network so as to minimize the total demand covered subject to the condition that no two facilities are allowed to be closer than a pre-specified distance. Drezner, Drezner and Scott (2009) analyzed the location of a facility inside a planar network with nuisance/hazard created on its links, so the total nuisance should be minimized. Lately, Yamaguchi (2011) examined a line network model where individuals collectively choose the location of an undesirable public facility through bargaining with the unanimity rule.

Regarding surveys and reviews on undesirable location, Moon and Chaudhry (1984) discussed and surveyed uncapacitated distance constrained network location problems such as maxian, defense, anti-center, dispersion, anticenter-maxian and dispersion-defense models. A widely cited review on this subject was due to Erkut and Neuman (1989), who brilliantly surveyed over sixty papers on maximization location models and presented a synthesis of the solution methods. In the same sense, Erkut and Verter (1995), and later Verter and Erkut (1995), overviewed and treated logistics models involving hazardous materials.

In addition to the network models, it is also worth citing some papers due to their contribution and direct application. One of these papers is the overview on (semi-) undesirable facility location by Plastria (1996). A close related paper by Carrizosa and Plastria (1999) presented a critical overview of the mathematical models used in the field of semi-obnoxious facility location. Murray, Church, Gerrard and Tsui (1998) reviewed several approaches for addressing equity and community impact in the location of undesirable facilities. In an excellent report, Cappanera (1999) surveyed mathematical models for undesirable location problems in the plane and particularly on networks.

Concerning straightforward applications, we must cite Cáceres, Mesa and Ortega (2007), who considered the problem of locating a waste pipeline in a coastal region, taking into account the maximization of two criteria.

There is no book solely devoted to location of undesirable facilities yet. Daskin (1995) discussed dispersion models, outlined a maxisum problem and commented on some multiobjective location problems. In Puerto (1996), there is a chapter concerning location of undesirable centers on the plane as well as on networks.

The two latest book chapters on undesirable facility location are by Hosseini and Esfahani (2009), who reviewed obnoxious facility location problems from the point of view of their classification, diverse models, applications, solutions and techniques, and case studies; and by Melachrinoudis (2011), who surveyed and assessed the classical contributions on undesirable facility location from the late 1970s till nowadays.

In the next section we review the most relevant references on multiobjective/multicriteria undesirable facility location models on networks.

3.3 Multicriteria undesirable facility location on networks

Surprisingly, literature on multicriteria undesirable facility location starts in the late 1980s. It seems that the concern on the location of undesirable facilities has grown only in the last years, along with the use of multiobjective/multicriteria tools to model and solve such problems.

Ratick and White (1988) proposed a multiobjective model for the location of undesirable facilities considering three objectives: minimizing the facility location costs, minimizing the opposition to the siting plan, and maximizing equity. List and Mirchandani (1991) presented a combined routing/siting model that can be used not only for making routing decisions on waste shipments, but also for siting decisions of waste treatment facilities. Risk, cost and risk equity were considered jointly in a multiobjective framework. A simplified form of their model was applied to the Capital District of the State of New York. Erkut and Neuman (1992) developed a multiobjective model for the location of one or more undesirable facilities to service a region which minimizes the total cost of the facilities located, the total opposition to such facilities, and the number of power-generating stations.

By means of a multiobjective model, Rahman and Kuby (1995) examined the tradeoffs between minimizing costs (transshipment and fixed-charge problems) and public opposition (decreasing distance function from the facility) in the location of a solid waste transfer station. A case study was also accomplished in the City of Phoenix, Arizona.

Giannikos (1998) presented a multiobjective model for locating disposal facilities and transporting hazardous waste along the links of a network considering four objectives, namely, minimization of total operating cost, minimization of total perceived risk, equitable distribution of risk among population centers and equitable distribution of the disutility caused by the operation of the treatment facilities.

Zhang and Melachrinoudis (2001) considered the problem of locating an obnoxious facility on a general network using two objectives, maximizing the minimum weighted distance from the point to the vertices (maximin) and maximizing the sum of weighted distances between the point and the vertices (maximsum). Hamacher, Labbé, Nickel and Skriver (2002) presented a polynomial time algorithm for the location of a semi-obnoxious facility on networks, and generalized the results to include maximin and minimax objectives.

Skriver and Andersen (2003) modeled a semi-obnoxious facility location problem as a bicriterion problem in both the plane and the network case, applying these models to the location of a new international airport in the Jutland mainland, Denmark.

Colebrook and Sicilia (2007) analyzed several location problems of undesirable facilities on multicriteria networks establishing new properties to characterize the efficient solutions and rules to remove inefficient edges. Tuzkaya, Önüt, Tuzkaya and Gülsün (2008) addressed the problem of locating an undesirable facility in Istanbul (Turkey) using the multi-criteria decision making technique called Analytic Network Process (ANP). Lately, Zhao and Shuai (2010) proposed a new multi-objective 0-1 integer LP model for the location-allocation problem in response network design for hazardous materials transportation.

Once more, the ensuing papers are commented for their real life application, though they might not be addressed on networks. Melachrinoudis, Min and Wu (1995) developed a dynamic (multiperiod) multiobjective mixed integer programming model for locating landfills. Their objectives are: minimization of total cost during the planning horizon, minimization of total risk posed on population centers, minimization of total risk posed on ecosystem and minimization of risk inequity over all individuals and time periods in the planning horizon.

Hokkanen and Salminen (1997) described an application of multicriteria decision aid to the location of a waste treatment facility in eastern Finland. The alternative locations for the new facility were considered based on 14 criteria by 28 decision makers.

Rakas, Teodorović and Kim (2004) developed a multiobjective model to determine the location of undesirable facilities using real-world data. Alumur and Kara (2007) proposed a new multiobjective hazardous waste location-routing model that minimizes the total cost and the transportation risk, and it was implemented in the Central Anatolian region of Turkey.

To the best of our knowledge, there is no published book on multicriteria undesirable facility location problems on networks. However, Daskin (1995) devoted a complete section of a chapter to emphasize the need of more multicriteria models on undesirable facility location.

Lastly, before presenting some basic definitions and the notation, we briefly comment four doctoral dissertations on multicriteria undesirable location. Saameño (1992) studied the problem of locating obnoxious facilities on a polygonal region with multiple objectives. Zhang (1996) mainly developed algorithms to solve the 1-maximin problem on a network, and the maximin-maxisum network location problem. Skriver (2001) investigated, among other models, the bicriterion semi-obnoxious location problem, the multicriteria semi-obnoxious network location problem with sum and center objectives and the bicriteria network location problem with criteria dependent lengths and minisum objectives. Finally, Colebrook (2003) devoted several chapters to analyze and develop some undesirable location models on networks.

4. Basic definitions and notation

In this section we introduce the concepts and basic definitions that are essential for the remaining sections. We begin with the notation on classical network models, followed by the definitions related to networks with multiple criteria.

4.1 Standard networks

Mathematical networks can model innumerable real world problems such as aisle/road networks, river/air/ocean transport networks or communication/computer networks. All of these networks are barring exceptions, simple (no loops or multiple edges), connected and undirected.

Thus, let $N = (V, E)$ be a network with such features, where $V = \{v_1, v_2, \ldots, v_n\}$ denotes the set of vertices or nodes, and $E = \{(v_s, v_t) : v_s, v_t \in V\}$ the set of edges, with $n = |V|$ and $m = |E|$. The nodes represent demand, supply or junction points on which existing facilities or clients are already placed, whereas edges correspond to transportation lines, roadways, railways or communication channels.

Each node $v_i \in V$ is set with a positive weight w_i as follows:

$$\begin{array}{rccc} w: & V & \longrightarrow & \mathbb{R}_+ \\ & v_i \in V & \longrightarrow & w(v_i) = w_i > 0 \end{array}$$

This weight w_i stands for demand rates, time/cost/loss per unit distance, number of clients, probability that a demand occurs at node v_i, or even the importance of a potential damage. Obviously, the weights are positive because a weight $w_i = 0$ means null demand, time, etc, and hence it makes no sense.

On the other hand, each edge $e = (v_s, v_t)$ is labeled with a positive number l_e in terms of the following length function:

$$\begin{array}{rcl} l: \quad E & \longrightarrow & \mathbb{R}_+ \\ e=(v_s,v_t)\in E & \longrightarrow & l(e)=l_e>0 \end{array}$$

Thus, a point *x* inside edge *e* ranges in the interval $[0,l_e]$. This length represents travel time/cost, reliability or any other travel attribute. The lengths are positive since any $l_e=0$ implies a null distance between v_s and v_t, and hence, it can be discarded. Fig. 1 shows a network with $n=5$ nodes and $m=7$ edges. Weights w_i are in bold, whereas lengths l_e are in italic.

Besides, each edge is assumed to be rectifiable, in the sense that there is a one-to-one correspondence between each edge and the interval $[0,1]$. Hence, given any edge $e=(v_s,v_t)\in E$ of length l_e and an inner point $x\in e$, then there is a unique number $t_e(x)\in[0,1]$ such that $t_e(x)l_e$ and $(1-t_e(x))l_e$ are the lengths along edge *e* between v_s and *x*, and *x* and v_t, respectively.

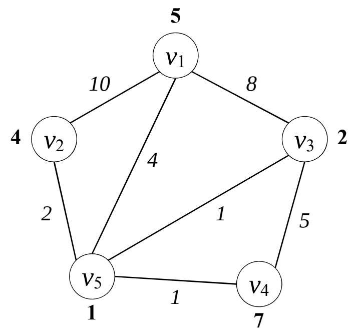

Fig. 1. Network with five nodes (weights in bold) and seven edges (lengths in italic).

A *path* is a sequence of adjacent edges, with each of the adjacent edges sharing a common node. Then, for each pair of nodes $v_a,v_b\in V$ we define a *distance* $d(v_a,v_b)$ between these two nodes as the length of any shortest path in *N* joining v_a and v_b. Moreover, given any two points $x,y\in N$, the distance $d(x,y)$ is the length of the shortest path between *x* and *y*. Given a certain edge $e=(v_s,v_t)$, it is sometimes possible that $d(v_s,v_t)<l_e$ since the edge may not provide the shortest path between the nodes v_s and v_t. This distance function $d(\cdot,\cdot)$ satisfies the following *metric properties* for any $x,y\in N$:

1. *Nonnegativity*: $d(x,y)\ge 0$, with $d(x,y)=0$ if $x=y$.
2. *Symmetry*: $d(x,y)=d(y,x)$.
3. *Triangle inequality*: $d(x,y)\le d(x,z)+d(z,y)$, for any $z\in N$.

At this point, the principal issue to be emphasized is that network location models are usually based on the assumption that travel distances are lengths of shortest paths. In this sense, given any edge $e = (v_s, v_t) \in E$, a node $v_i \in V$ and an inner point $x \in e$, we define the distance between point *x* and node v_i as:

$$d(x, v_i) = \min\{x + d(v_s, v_i), l_e - x + d(v_t, v_i)\}$$

The point on *e* where $d(x, v_i)$ attains its equilibrium, i.e. $x + d(v_s, v_i) = l_e - x + d(v_t, v_i)$, is called a *bottleneck point* b_i, with

$$b_i = \frac{d(v_t, v_i) + l_e - d(v_s, v_i)}{2}$$

A fundamental property of network distances is the following *piecewise linearity and concavity property*. This property states that the function in $x \in e = (v_s, v_t)$ defined by $d(x, v_i)$:

1. Is continuous on *e*.
2. As *x* varies from node v_s to v_t in edge *e*, either
   - increases linearly with slope w_i (see Fig. 2a), or
   - decreases linearly with slope $-w_i$ (see Fig. 2b), or
   - first increases linearly and then decreases linearly, with a breakpoint at b_i (see Fig. 2c).
3. Is *concave*, in the sense that a line segment joining any two points on the graph of the function lies on or below such graph.

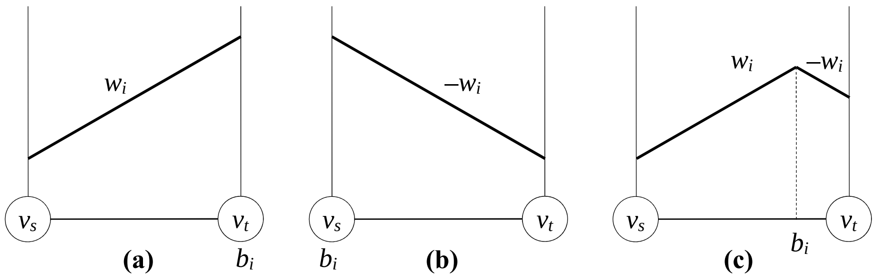

Fig. 2. The three possible plots of $d(x, v_i)$.

These are the basic concepts on standard networks. In the next section we introduce the basic notions on networks with multiple criteria, namely, considering several weights on each node as well as several lengths on each edge.

4.2 Networks with multiple parameters on nodes and edges

Most of the huge literature on network location problems deals with the optimization of one *single criterion*. This criterion is usually associated with the weighted distance from a certain point to the rest of the nodes, for example, the minimization of the total weighted distance from a facility to the customers.

However, there are many applications in which several parameters need to be considered on each node and on each edge. Thus, several weights on each node may represent different criteria to be considered by the decision-maker(s), namely, demand rate, importance, number of potential clients, etc. On the other hand, several lengths (travel costs) on each edge might deal with distance, travel time, traffic congestion, toll rate, travel cost, etc.

In this sense, on each node $v_i \in V$, the previous weight function is now replaced by the following:

$$\begin{array}{rcl} w: \quad V & \longrightarrow & \mathbb{R}^p \\ v_i \in V & \longrightarrow & w(v_i) = w_i = (w_i^1, \ldots, w_i^p) \end{array}$$

where p is the number of weights per node. For any vector of weights w_i, each w_i^r is a nonnegative value for $r = 1, \ldots, p$, and we assume that not all are equal to zero.

Likewise, each edge is set with a vector of lengths (costs), as follows:

$$\begin{array}{rcl} l: \quad E & \longrightarrow & \mathbb{R}^q \\ e = (v_s, v_t) \in E & \longrightarrow & l(e) = l_e = (l_e^1, \ldots, l_e^q) \end{array}$$

in which q is the number of lengths. Again, we assume that each component l_e^r is nonnegative for any vector l_e, and not all $l_e^r = 0$, for $r = 1, \ldots, q$.

As an example of a network holding several parameters, Fig. 3 shows the same network as Fig. 1, but with two weights per node (in bold) and three lengths per edge (in italic).

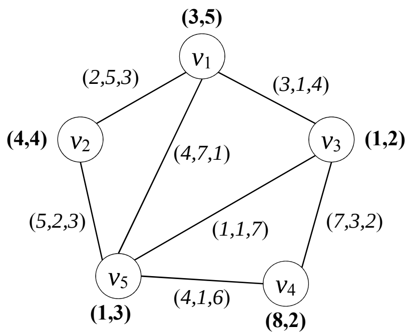

Fig. 3. Five-node and seven-edge network with several parameters.

Let *r* be a length index, with $1 \le r \le q$, and let $x \in e = (v_s, v_t)$ be a point inside edge *e*. Then, $c_e^r(x, v_s)$ is defined as the piece of line segment between *x* and v_s considering length *r*. Obviously, we have that $0 \le c_e^r(x, v_s) \le l_e^r$, with $c_e^r(x, v_t) = l_e^r - c_e^r(x, v_s)$.

For each pair of nodes $v_a, v_b \in V$ we can define the *distance* $d^r(v_a, v_b)$ between these two nodes as the length of any shortest path in *N* joining v_a and v_b considering length *r*. Likewise, given any two points $x, y \in N$, the distance $d^r(x, y)$ is the length of the shortest path between *x* and *y*. These *q* distance functions also comply with the metric properties stated in the preceding section.

Given any node $v_i \in V$, we have that

$$d^r(x, v_i) = \min\{c_e^r(x, v_s) + d^r(v_s, v_i), c_e^r(x, v_t) + d^r(v_t, v_i)\}$$

denotes the distance between a point and a node considering length *r*, with $b_i^r = (d^r(v_t, v_i) + l_e^r - d^r(v_s, v_i))/2$ being the bottleneck point concerned with node v_i. These *q* network distance functions fulfill the piecewise linearity and concavity property as well.

Finally, we introduce some basic theory on multicriteria/multiobjective optimization. Usually, *multicriteria* models are those which perform a simultaneous optimization of several incommensurable objectives, for instance, minimizing the maximal travel distance and minimizing the total travel cost. On the other hand, a closely related concept is that of *vector optimization*, which determines the non-dominated solutions to a multicriteria problem.

In this sense, let $f = (f_1, f_2, \ldots, f_k)$ and $g = (g_1, g_2, \ldots, g_k)$ be two vectors belonging to $\mathbb{R}^k$. Vector *f* is said to *dominate* vector *g*, and it is denoted by $f \prec g$, if and only if:

$$f_i \le g_i, \forall i = 1,\dots,k \quad \text{and} \quad \exists j \in \{1,\dots,k\} : f_j < g_j$$

Then, given the subset of vectors $U \subseteq \mathbb{R}^k$, a vector $f \in U$ is called *non-dominated*, *efficient* or *Pareto optimal* (Pareto, 1896) with respect to subset U if there is no other vector $g \in U$ such that $g \prec f$. The set of all non-dominated vectors with respect to U is denoted by U_{ND}. For a further knowledge in multicriteria optimization, the reader is referred to Steuer (1986).

Having described the basic concepts and the notation used to model the location problems developed in this chapter, in the following sections we present the location models for undesirable facilities on networks.

5 Locating undesirable facilities on simple networks

In the following subsections we develop several models that can be used to locate hazardous facilities on networks considering one single criterion. These models comprise the undesirable center problem, the maxian problem, and the anti-cent-dian problem.

5.1 The undesirable center problem

As we remarked in the literature review, there are not many papers devoted to undesirable location on networks. One of them is by Melachrinoudis and Zhang (1999), who proposed a $O(mn)$ time algorithm based on three upper bounds and on a modified procedure of Dyer (1984). However, their upper bounds can be tightened, and the procedure can be improved by means of a more convenient formulation of the solution. The other paper by Berman and Drezner (2000) approaches the problem in a linear programming way. Though it has the same theoretical complexity, its running time is extremely high, since the algorithm has to process every single edge.

Now, we formulate the undesirable 1-center (maximin) problem on networks. Given any point $x \in N$ we define $f(x) = \min_{v_i \in V} w_i d(x, v_i)$.

Then, the problem consists of calculating

$$\max_{x \in N} \min_{v_i \in V} w_i d(x, v_i) = \max_{x \in N} f(x)$$

and a point $x_N \in N$ is an undesirable 1-center point iff $f(x_N) = \max_{x \in N} f(x)$.

This problem is the opposite of the 1-center problem (minimax), so it could be called the *anti-center*. Unfortunately, this term was already coined by Minieka (1983) to define the *maxmax* problem. We instead propose the term 1-*uncenter* (undesirable center) to define the optimal location point.

If there is at least one vertex v_i such that $w_i = 0$, then $f(x) = 0, \forall x \in N$ and obviously any point on network *N* would be a 1-uncenter. Therefore, we consider only $w_i > 0$, $\forall v_i \in V$.

When all the node weights are equal, $\forall v_i \in V$, $w_i = w$, the local 1-uncenter x_e is sited at the central point of edge *e*. Therefore, the unweighted 1-uncenter x_N is located in the middle of the longest edge(s) (see Melachrinoudis and Zhang, 1999; Berman and Drezner, 2000). This is done in $O(m)$ time.

However, when all node weights are not equal, we can reformulate the 1-uncenter problem over each edge $e = (v_s, v_t) \in E$ as follows: $x_N \in N$ is a 1-uncenter point iff $f(x_N) = \max_{e \in E} f(x_e)$.

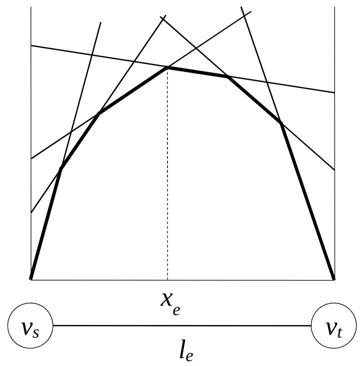

Fig. 4. Objective function $f(x)$, which is actually the lower envelope of all distance functions.

Since the local 1-uncenter point is the maximum value of the concave objective function $f(x)$, as shown in Fig. 4, it should be located at the intersection of two distance functions lines with opposite sign slopes. Our goal is to find in the lower envelope of function $f(x)$ these two lines and the intersection point between them.

By introducing new tighter bounds that can significantly reduce the number of edges and the number of distance function lines over each edge, and by means of a more convenient problem formulation, we developed a new $O(mn)$ time algorithm, which is briefly outlined in Algorithm 1.

This method has been applied to the following network depicted in Fig. 5, which has $n = 8$ nodes and $m = 18$ edges. The weights (in bold) on the nodes range randomly from 1 to 9, whereas the lengths (in italics) randomly vary from 1 to 49.

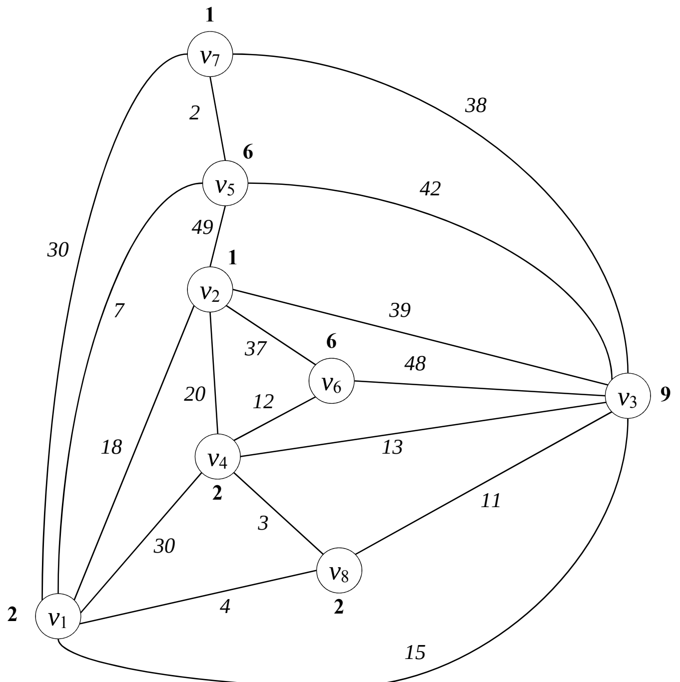

Fig. 5. Planar network with $n = 8$ and $m = 18$.

The solution to this example is $F_N = 50$, which is the 1-uncenter value at $S = \{(26, e_{36})\}$. Note that the algorithm processes only 6 out of 18 potential edges. Even though these numbers may not seem important, they will be quite relevant when the network size gets bigger, both in nodes and edges.

To test the computational effort of the new algorithm, several experiments were run for different sets of graph densities as well as for planar networks. These tests showed that the running times of the new algorithm are faster than both the approaches by Berman and Drezner (2000) and Melachrinoudis and Zhang (1999), since the number of edges processed are less, gaining in some instances a reduction of over 50%. As a consequence, the computing times of the new algorithm are better, achieving in some cases a reduction of 80%. Besides, the reduction augments as the number of nodes *n* increases.

For more details in the mathematical results, algorithm description, the example trace and the computational time experiment, the reader is referred to Colebrook *et al.* (2002).

```
function UnCenter(Network N, Distance Matrix d)
{   // Current best value on network N
    F_N := 0
    // Solution set
    S := ∅
    for all edges e := (v_s, v_t) ∈ E do
    {   // Compute the upper bounds
        Compute upper bound UB1
        if F_N > F_UB1 then continue to next edge
        Compute upper bound UB2
        if F_N > F_UB2 then continue to next edge
        Compute upper bound UB3
        if F_N > F_UB3 then continue to next edge
        // Set (x_e, F_e) to the best value found so far
        if F_UB2 ≤ F_UB3 then   (x_e, F_e) := (x_UB2, F_UB2)
        else                    (x_e, F_e) := (x_UB3, F_UB3)
        Create sets L and R. All lines must be below F_UB2 .
        // Continue till the new value F_e cannot improve the current F_N ,
        // or until one of the node sets becomes empty
        while F_e ≥ F_N and ( L ≠ ∅ or R ≠ ∅ ) do
        {   Pair all nodes in L against R, using a max{|L|,|R|} matching
            (x_e, F_e) := Intersection point with minimal function value
            Project the value x_e on the lower envelope to get v_a and v_b
            x_e := Intersection point of distance lines v_a and v_b
            F_e := Distance value of point x_e
            Remove from L and R all lines above the new value F_e
        }
        if F_e ≥ F_N then
        {   F_N := F_e
            Store the pair (x_e, e) in S
        }
    }
    return (F_N, S)
}
```

Algorithm 1. The uncenter function.

5.2 The maxian problem

As stated in the review section, the literature on undesirable network location began in the mid 70s with Church and Garfinkel (1978), who defined and solved the 1-maxisum (*maxian*) problem in $O(mn\log n)$ time, being *n* the number of nodes and *m* the number of edges.

Later on, Tamir (1991) briefly suggested that the 1-maxisum problem could be solved in $O(mn)$ time using an algorithm given by Zemel (1984). However, to the best of our knowledge, there is no reference in the literature directly describing such an algorithm for the network 1-maxisum problem thus far. Hence, in this section we provide an algorithm which solves this problem in $O(mn)$ time.

Given any point *x* on network *N*, we define

$$f(x) = \sum_{v_i \in V} w_i d(x, v_i)$$

as the sum of weighted distances from point *x* to all the nodes of the network.

The undesirable *one-facility maximum* (maxian) *problem* is expressed as

$$\max_{x \in N} f(x)$$

and a point $x_N \in N$ is a *maxian* point iff $f(x_N) = \max_{x \in N} f(x)$. Several interesting properties arise for this problem.

From Church and Garfinkel (1978), an initial upper bound $UB(e)$ is derived, which is improved with a new upper bound. Likewise, this bound can be dynamically updated without increasing the total computational time. Hence, we have developed a new algorithm in $O(mn)$ to solve this problem. The procedure makes use of the new upper bound, and thus, allows skipping out from the search process as soon as the upper bound is less than the global optimum. The outline of the new procedure is showed in Algorithm 2.

To illustrate the method, consider the network in Fig. 6 with $n = 7$ nodes and $m = 15$ edges. The node weights (in bold) are integers randomly generated between 1 and 9, whereas the edge lengths range between 1 and 25.

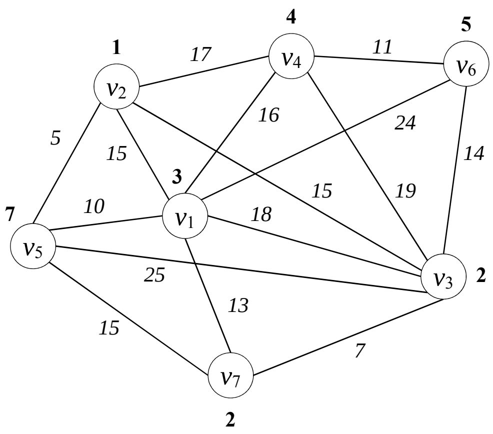

Fig. 6. Weighted network with seven nodes and fifteen edges.

The optimum value of $f(x)$ in this example is $f_N = 500$, which is attained in the interval [8.5,10.5] at edge (v_3, v_4). We finally remark that, due to the new upper bound, we have only processed 8 of the 15 total edges. Using the old upper bound the algorithm would have run over 13 edges. This fact is very important, since it speeds up the search for the optimal points once we are sure that the new upper bound is worse thant the current best solution.

This new algorithm has been compared with the procedure proposed by Church and Garfinkel (1978), including the initial bound, on low and high dense networks, as well as on planar networks. In all cases, the new algorithm accomplishes a better performance in terms of processing times. The computational experiment was tested on complete networks with, respectively, a half, a quarter and an eighth of the total number of edges. In the three cases, the new algorithm is almost 50% faster than Church and Garfinkel's. The same happens for planar networks. Besides, the reduction percentage in the number of edges to be processed is almost 25% less.

Again, we refer the reader to Colebrook, M., Gutiérrez, J. and Sicilia, J. (2005) for the details of this algorithm and the new upper bound.

5.3 The anti-cent-dian problem

In previous sections we have addressed the 1-uncenter (*maximin*) problem and the 1-maxian (*maxisum*) problem on networks. Now we are going to combine these two objectives to obtain a location criterion called the *anti-cent-dian*.

function *MaxianAlgorithm*(Network *N*, Distance Matrix *d*)
{ $f_N := 0$ *// Current best value on network N*
 $S := \emptyset$ *// Solution set*
 for all edges $e := (v_s, v_t) \in E$ **do**
 { Compute W_s and W_t
  $X_e := \emptyset$
  // Let X_e represent either a single point x or an interval $[x^1, x^2]$.
  if (W_s and W_t yield a simple solution) **then** Store solution in X_e
  else
  { $F_j := f(v_s)$, $W_j := W_s$
   $F_k := f(v_t)$, $W_k := W_t$
   // Compute initial value of the new upper bound $NUB(e)$
   if $NUB(e) < f_N$ **then** continue to next edge
   $l := 1$, $r := n$
   while $X_e = \emptyset$ **and** $NUB(e) \ge f_N$ **do**
   { $d_q :=$ Median value of all d_i with $l \le i \le r$
    $b_q := (d_q + l_e) / 2$
    Compute W_L and W_R
    if (W_L, W_R and w_q yield a solution) **then** Store it in X_e
    else
    { *// Search for the optimum to the left or right*
     if $W_L + w_q < W_R$ **then**
       $l := q + 1$, update F_j, W_j, W_L, $f(b_q)$
     else $r := q - 1$, update F_k, W_k
     Update the upper bound $NUB(e)$ at point b_q
    }
   }
  }
  if $X_e \ne \emptyset$ **and** $f(X_e) \ge f_N$ **then**
  { $f_N := f(X_e)$
   Store the pair (X_e, e) in *S*
  }
 }
 return (f_N, S)
}

Algorithm 2. The new algorithm for the maxisum problem.

The network anti-cent-dian model considers the convex combination of the maximin and the maxisum criteria. Moreno-Pérez and Rodríguez-Martín (1999) developed two algorithms that provide, respectively, the optimal location for a fixed λ that determines the convex combination, and the set of optimal locations for all convex combinations. Both of them run in $O(mn\log n)$ time. In this section we show that the complexity of the first algorithm can be reduced to $O(mn)$.

We now define the unweighted uncenter (maximin) function and the maxian (maxisum) function. Given any point *x* on network *N*, we define

$$f_{\min}(x) = \min_{v_i \in V} d(x, v_i)$$

as the minimum unweighted distance from point *x* to all nodes of the network. Recall that a point $y_N \in N$ is an uncenter point iff $f_{\min}(y_N) = \max_{x \in N} f_{\min}(x)$. When all node weights w_i are equal, the point y_N is located in the middle of the longest edge. Then, the uncenter point for any edge $e = (v_s, v_t)$ is $y_e = l_e / 2$, and hence $f_{\min}(y_e) = l_e / 2$. Thus, the local optimum can be obtained in $O(1)$.

On the other hand, given $W = \sum_{v_i \in V} w_i$ and a point $x \in N$, we now define

$$f_{\text{sum}}(x) = \frac{1}{W} \sum_{v_i \in V} w_i d(x, v_i)$$

as the average sum of weighted distances from point *x* to all the nodes of the network. A point $z_N \in N$ is a maxian point iff $f_{sum}(z_N) = \max_{x \in N} f_{sum}(x)$. The local maxian point on edge *e* is denoted by z_e.

Finally, the *anti-cent-dian* function is defined as

$$f_{\text{acd}}(\lambda, x) = \lambda f_{\min}(x) + (1 - \lambda) f_{\text{sum}}(x)$$

and any point $x_N \in N$ maximizing $f_{\text{acd}}(\lambda, x)$ for a particular value of λ, $0 \le \lambda \le 1$, is called a *λ-anti-cent-dian* point. In particular, if $\lambda = 0$, the anti-cent-dian is equal to the maxian; whereas for $\lambda = 1$, we obtain the uncenter. Fig. 7 shows a typical plot of function $f_{\text{acd}}(\lambda, x)$ over edge *e*. For $\lambda = 0$ the anti-cent-dian function is $f_{\text{sum}}(x)$. As parameter λ grows, the anti-cent-dian function makes a *morphing* to the $f_{\min}(x)$ function.

Taking into account some properties and given a value of λ, $0 \le \lambda \le 1$, the latter problem can be formulated over each edge *e* as follows:

$$f_{\text{acd}}(\lambda, x_e) = \max_{x \in e} f_{\text{acd}}(\lambda, x)$$

and a point $x_N \in N$ is a λ-anti-cent-dian point iff $f_{\text{acd}}(\lambda, x_N) = \max_{e \in E} f_{\text{acd}}(\lambda, x_e)$.

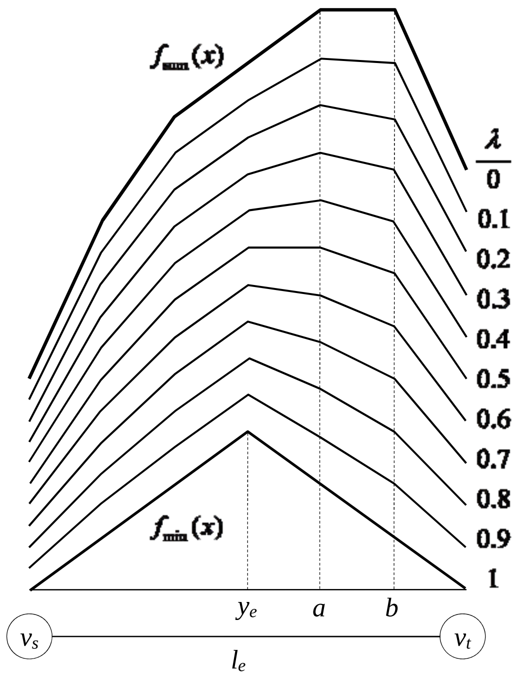

Fig. 7. Plots of $f_{\text{acd}}(\lambda, x)$ for different values of λ.

A method to determine all λ-anti-cent-dian points for any value of $\lambda \in [0,1]$ in $O(mn \log n)$ time was proposed by Moreno-Pérez and Rodríguez-Martín (1999). It has derived from an $O(mn \log n)$ algorithm by Hansen, Labbé and Thisse (1991). This complexity cannot be reduced since the algorithm is based on the computation of a convex hull of $O(mn)$ points, which is done in $O(mn \log n)$ time (see Hershberger, 1989).

On the other hand, Moreno-Pérez and Rodríguez-Martín (1999) also presented an $O(mn \log n)$ procedure to obtain the anti-cent-dian point when λ is fixed to a particular value. Nevertheless, an $O(mn)$ time algorithm can be achieved, as shown in Colebrook and Sicilia (2006).

Since the following multicriteria location model generalizes the λ-anti-cent-dian problem described above, the algorithm scheme and the example for this model are shown in the next section.

6 Undesirable facility location on multicriteria networks

As we have stated in the preceding sections, the classical location criteria *minimax* (center) and *minisum* (median) are useless to locate an *obnoxious/noxious* (undesirable) facility. Thus, the *maximin/maxmax* and the *maxisum* criteria arose to model, respectively, the undesirable center problem and the undesirable median problem. By placing the new facility away from existing facilities, the maximin criterion reduces the effect on the worst impacted existing facility, whereas the maxisum criterion diminishes the collective effect (average) on the existing facilities.

Nevertheless, some facilities might be considered *semi-desirable* since they provide a main service to the community but they can also cause inconveniences to the neighboring areas, for instance, an airport, a train station, or any other noisy facility. These problems can be perfectly modeled combining the minimax/minisum criteria and the maximin/maxisum criteria.

In this sense, most of the undesirable facility location models analyzed in previous works are basically single-criterion. However, Erkut and Neuman (1989) emphasized on the need for multiobjective approaches to the siting of undesirable facilities. Daskin (1995) and Zhang (1996) also pointed out not only the need to include multiple criteria in undesirable facility location problems, but also the fact that poor attention has been paid by researchers to these problems and hence, little research has been done in this promising field.

Accordingly, in this section we present a multicriteria undesirable facility location model on networks with several weights on the nodes and several lengths on the edges, combining the maximin and maxisum criteria by a parameter λ. Such a model can be considered as the opposite to the multicriteria network λ-cent-dian problem presented in the last section and hence, it can be described as the *multicriteria λ-anti-cent-dian* problem on networks.

Given any point $x \in N$, any weight s ($1 \le s \le p$) and any length r ($1 \le r \le q$), let $f_{\min}^{sr}(x) = \min_{v_i \in V} w_i^s d^r(x, v_i)$ be the minimum weighted distance from x to the set of nodes. Besides, given any point $x \in N$, we define the function $f_{\text{sum}}^{sr}(x) = \sum_{v_i \in V} w_i^s d^r(x, v_i)$ as the sum of weighted distances from point x to the set of nodes, with $1 \le s \le p$ and $1 \le r \le q$.

Through a parameter λ, the convex combination of these two latter problems was addressed as the multicriteria λ-anti-cent-dian problem. Thus, given $\lambda \in [0,1]$ and $x \in N$, the λ-anti-cent-dian function is defined as follows

$$f_{\text{acd}}^{sr}(\lambda, x) = \lambda f_{\min}^{sr}(x) + (1-\lambda) f_{\text{sum}}^{sr}(x)$$

being $f_{\min}^{sr}(x)=\min_{v_i\in V} w_i^s d^r(x,v_i)$ and $f_{\text{sum}}^{sr}(x)=\sum_{v_i\in V} w_i^s d^r(x,v_i)$, with $s=1,\ldots,p$ and $r=1,\ldots,q$. This model was introduced in a previous section, though function $f_{\min}(x)$ was unweighted and $f_{\text{sum}}(x)$ was divided by the total sum of weights. Provided that both $f_{\min}^{sr}(x)$ and $f_{\text{sum}}^{sr}(x)$ are continuous, concave and piecewise linear functions on *x*, the λ-anti-cent-dian function $f_{\text{acd}}^{sr}(\lambda,x)$, being a convex combination of the two latter functions, fulfills these characteristics as well.

As shown in Algorithm 3, we proposed a rule to delete inefficient edges and a polynomial algorithm in $\mathrm{O}(k^3m^2n^2)$ time to solve this problem, being *k* the number of criteria. Besides, for $\lambda=0$ we can solve the multicriteria maxian problem, whereas for $\lambda=1$ we can obtain the solution for the multicriteria uncenter problem. Furthermore, when $p=q=1$ this procedure can even solve the single criterion uncenter, maxian or anti-cent-dian problem. The computational experience strengthens the polynomial complexity of the algorithm as well as the effectiveness of the rule to eliminate the inefficient edges.

To illustrate the method, Fig. 8 shows a random planar network with $n=7$ nodes, $m=15$ edges, $p=2$ weights per node and $q=2$ lengths per edge. Thus, we have $k=4$ criteria. Beside each node $v_i\in V$ we placed (in bold) two integer weights (w_i^1,w_i^2) randomly generated in the interval $[1,5]$. Likewise, each edge $e=(v_s,v_t)\in E$ is labeled (in italics) with two integer lengths (l_e^1,l_e^2) randomly ranging in the interval $[1,25]$. We set the parameter λ to 0.5.

The algorithm begins by removing all edges that contain no efficient point. For the example shown in Fig. 8, only 8 out of the 15 initial edges remain after the deletion, namely: (v_1,v_3), (v_1,v_4), (v_2,v_5), (v_2,v_6), (v_3,v_4), (v_3,v_5), (v_4,v_5) and (v_5,v_6). On this set of remaining edges we now proceed to compute, for each combination of weights and lengths, the functions $f_{\min}^{sr}(x)$ and $f_{\text{sum}}^{sr}(x)$. Subsequently, given the parameter $\lambda=0.5$ we calculate the λ-anti-cent-dian functions $f_{\text{acd}}^{sr}(\lambda,x)$.

Finally, the solution is the set of non-dominated segments, which are located on 5 edges only. The set of efficient points is shown in Table 1 and it is also drawn in bold on the partial network of Fig. 9.

```
function MACD(Network N(V,E), DistanceMatrix d, Parameters p, q, λ)
{   Let P := ∅ be the set of candidate points to be non-dominated
    Let S := ∅ be the set of possible non-dominated segments
    Remove all edges containing no efficient points
    for all remaining edges e := (v_s, v_t) ∈ E do
    {   for s := 1 to p do
            for r := 1 to q do
            {   if λ ≠ 0 then Compute f^{sr}_min(x)
                if λ ≠ 1 then Compute f^{sr}_sum(x)
            }
        for s := 1 to p do
            for r := 1 to q do
                Compute f^{sr}_acd(λ, x) = λ f^{sr}_min(x) + (1 − λ) f^{sr}_sum(x)
        Get the set of efficient points X_e
        Let x_1, ..., x_j be the sorted sequence of j breakpoints for the
            k = p × q λ-anti-cent-dian functions inside X_e
        if j = 1 then P := P ∪ {x_1}
        else
            for i := 1 to j − 1 do
            {   Let [x_i, x_{i+1}] be a segment of edge e within X_e
                S := S ∪ {[x_i, x_{i+1}]}
            }
    }
    // Let P_ND the set of non-dominated points and S_ND the
        set of non-dominated segments.
    P_ND := PointComparison(P)
    S_ND := SegmentComparison(S)
    (P_ND, S_ND) := PointAgainstSegmentComparison(P_ND, S_ND)
    return P_ND and S_ND
}
```

Algorithm 3. The multicriteria λ-anti-cent-dian function (MACD).

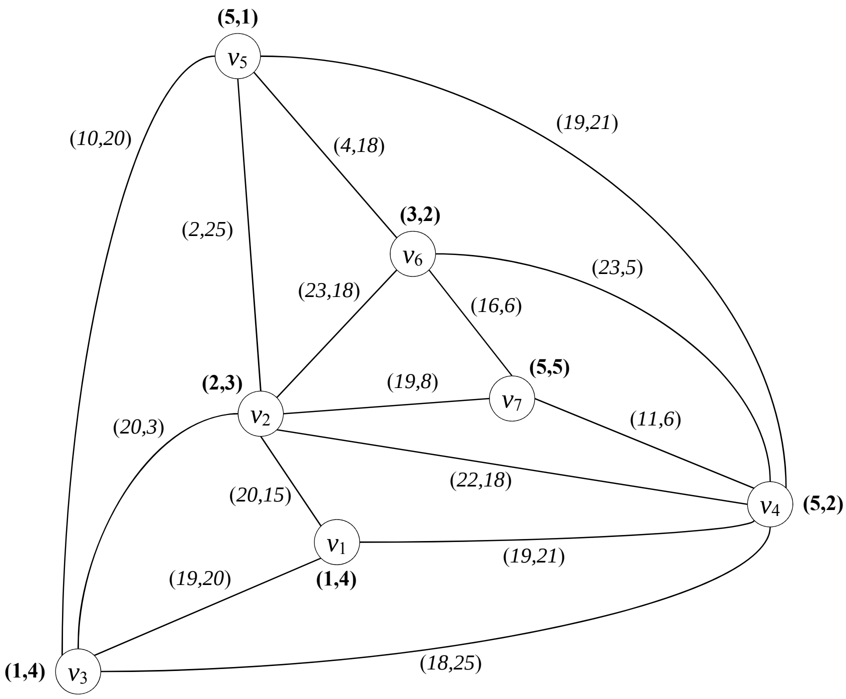

Fig. 8. A network with two lengths per edge and two weights per node.

Edge	Efficient points
(v_1, v_3)	[3.8,8.5]
(v_2, v_5)	[1.00923,1.64]
(v_2, v_6)	[9.61111,12.5]
(v_3, v_5)	[5.28571,8.25]
(v_4, v_5)	[7.9418,15.381]

Table 1. Set of efficient points of the network shown in Fig. 8.

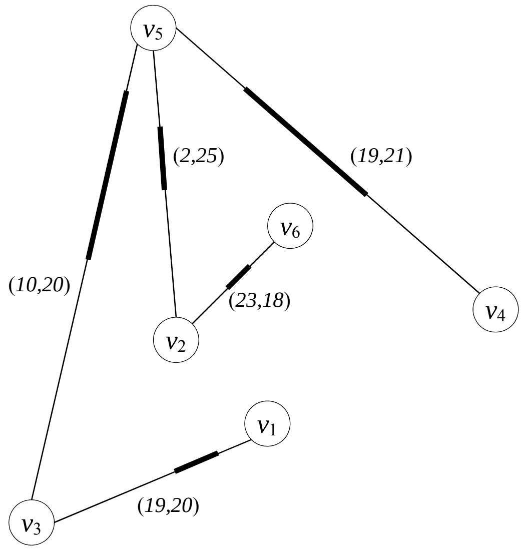

Fig. 9. Efficient points are drawn in bold on the partial network.

Algorithm 3 was programmed in C++ programming language using the class library LEDA 4.2.1, on a two 1.2 Ghz processor Pentium III with 1 Gb of RAM under Red Hat Linux.

Two kinds of experiments were performed. In both of them, random planar networks were generated with $m = 3n - 6$ edges using the generators developed by LEDA. Likewise, parameter λ varies from $\lambda = 0$ (maxian problem) to $\lambda = 1$ (uncenter problem) with a step of 0.5. Both the number of weights per node *p* and the number of lengths per edge *q* range between 1 and 3. Ten instances were generated for each combination of the latter parameters. The weight values are random integers uniformly distributed in the interval $[1,10]$, whereas the edge lengths are random integers in the range $[1,50]$. We remark that calculation of the distance matrix was not included in the total computing time.

In the first experiment, random planar networks were generated with $n = 10$ up to 100 in steps of 10 nodes. Table 2 shows the average times. Regardless of the number of nodes *n*, the computing time grows as both *p* and *q* increase. The average percentage of edges deleted is shown in Table 3. In most cases the number of removed edges is very high, achieving in some instances 99% of deletion. This issue becomes quite remarkable when $p = q = 1$ (single criterion). In this particular case, the bounds seem to be very tight, and thus, the removal rule becomes very effective since over 95% of the edges are deleted, leaving only those edges that contain the final optimal points.

Moreover, the performance of the new algorithm was also tested on bigger random planar networks with $n = 50$ to 500 nodes, with a step of 50 nodes. In the case of $p = q = 1$, the percentage of deletion in all cases is over 99%. However, when $p = q = 3$, the average edge removal percentage is greater for $\lambda = 0$ than for $\lambda = 1$, and hence, the average times in the latter are higher.

This model can be slightly modified to generalize other models studied in the literature. For instance, if we define a set of *k* parameters $\Lambda = \{\lambda^1, \ldots, \lambda^k\}$, then we can deal with each function $f^i_{\text{acd}}(\lambda^i, x)$ independently. Thus, the problem proposed by Zhang and Melachrinoudis (2001) might be denoted as $\max_{x \in N}(f^1_{\text{acd}}(\lambda^1, x), f^2_{\text{acd}}(\lambda^2, x))$, with $p = 2$, $q = 1$, $k = p \times q = 2$ and $\Lambda = \{\lambda^1 = 1, \lambda^2 = 0\}$. On the other hand, the multicriteria semi-obnoxious median problem presented by Hamacher, Labbé, Nickel, and Skriver (2002) can be formulated as $\max_{x \in N}(f^i_{\text{acd}}(\lambda^i, x), -f^j_{\text{acd}}(\lambda^j, x))$, with $p > 1$, $q = 1$, $\lambda^i = \lambda^j = 0$ and $i \in Q_1$, $j \in Q_2$, $|Q_1 \cup Q_2| = p$, $Q_1 \cap Q_2 = \varnothing$, being Q_1 the set of obnoxious objective functions, and Q_2 the set of desirable objective functions. Obviously, if $Q_2 = \varnothing$ then we get the multicriteria maxian problem discussed in this chapter.

Finally, we remark that if $p > 1$ and $q = 1$ then the number of criteria matches the number of weights per node, i.e., $k = p$. Besides, if $\lambda = 0$ then the number of breakpoints for all the *k* objective functions of a given edge is $\mathrm{O}(n)$, since all the $f^{s1}_{\text{sum}}(x)$ functions share the same breakpoints. Hence, the total number of segments to compare is $\mathrm{O}(mn)$. Therefore, the overall complexity of the algorithm is reduced to $\mathrm{O}(km^2n^2)$, which is the same complexity achieved by Hamacher, Labbé, Nickel, and Skriver (2002) for the location of a semi-obnoxious facility on networks with sum objectives.

For more details, the reader is referred to Colebrook and Sicilia (2007).

Conclusions and directions for further research

This chapter aimed to be a comprehensive compilation of references and methods dealing with undesirable facility location on networks. In this sense, more than 80 papers have been briefly commented, along with several models on undesirable single facility location on networks with multiple criteria that have been analyzed and described.

We first addressed the undesirable 1-center (uncenter) location problem on networks. By means of a more suitable problem formulation, a new $\mathrm{O}(mn)$ algorithm can be developed, which is more straightforward and computationally faster than the ones already reported in the literature. Besides, we have also analyzed the problem of locating an undesirable median (maxian) on a network, obtaining a new and better upper bound. We have briefly presented the idea of a new algorithm in $\mathrm{O}(mn)$ time to solve this problem.

Finally, we studied the uncenter and maxian problems on multicriteria networks, establishing new properties and rules to remove inefficient edges. We have also presented the multicriteria λ-anti-cent-dian model as a convex combination of the two latter problems through a parameter λ. An effective rule to remove edges containing inefficient points, as well as a polynomial algorithm in $O(m^2n^2k^3)$ time, being k the number of criteria. Besides, this procedure can solve both the multicriteria uncenter problem and the multicriteria maxian problem. Moreover, when the network holds a single weight per node and a single length per edge, this algorithm can efficiently solve the single criterion uncenter, maxian and λ-anti-cent-dian problems. Lastly, this model might be slightly modified to generalize other models presented in the literature.

Some directions for future research could be:

- Try to apply the undesirable location problems to real world applications, or redesign them to acquire the real details that are not covered in the models. A direct use could be any application involving GIS (*Geographic Information System*) technologies.
- Compile in a single software application all the models described in this chapter, along with the classical algorithms for *desirable* facility location problems. A first attempt of this project was presented in Colebrook, Alonso and Sicilia (2005).
- Expose all the algorithms developed so far as Web Services in the Internet, so they could be easily used from any computing device (PC, smartphone, tablet, etc). This is a nice project that we keep in mind a long time ago, and we hope to develop it shortly.

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		λ = 0									λ = 0.5								
		$p = 1$			$p = 2$			$p = 3$			$p = 1$			$p = 2$			$p = 3$		
n	m	$q = 1$	$q = 2$	$q = 3$	$q = 1$	$q = 2$	$q = 3$	$q = 1$	$q = 2$	$q = 3$	$q = 1$	$q = 2$	$q = 3$	$q = 1$	$q = 2$	$q = 3$	$q = 1$	$q = 2$	$q = 3$
10	24	0.01	0.01	0.04	0.00	0.01	0.09	0.00	0.02	0.14	0.01	0.02	0.03	0.01	0.01	0.07	0.01	0.03	0.05
20	54	0.01	0.02	0.09	0.01	0.03	0.09	0.01	0.03	0.17	0.01	0.03	0.08	0.02	0.03	0.26	0.02	0.08	0.27
30	84	0.02	0.04	0.10	0.02	0.06	0.17	0.03	0.05	0.34	0.02	0.03	0.22	0.03	0.05	0.47	0.03	0.12	0.17
40	114	0.02	0.04	0.16	0.03	0.08	0.30	0.05	0.06	0.35	0.03	0.08	0.31	0.04	0.12	0.44	0.05	0.09	0.65
50	144	0.04	0.06	0.24	0.05	0.11	0.29	0.06	0.14	0.21	0.05	0.08	0.39	0.06	0.12	0.50	0.07	0.22	0.55
60	174	0.04	0.07	0.34	0.06	0.13	0.45	0.07	0.14	0.40	0.06	0.12	0.54	0.07	0.17	0.78	0.08	0.27	1.14
70	204	0.05	0.08	0.26	0.06	0.13	0.42	0.09	0.18	0.62	0.05	0.18	0.50	0.08	0.26	0.91	0.11	0.27	1.30
80	234	0.05	0.10	0.33	0.08	0.16	0.64	0.10	0.21	0.99	0.07	0.16	0.65	0.10	0.23	0.83	0.13	0.45	1.44
90	264	0.09	0.14	0.43	0.12	0.23	0.64	0.15	0.29	0.65	0.10	0.19	0.76	0.14	0.35	1.47	0.18	0.54	1.81
100	294	0.09	0.17	0.58	0.13	0.25	0.73	0.17	0.34	1.08	0.12	0.23	0.62	0.16	0.39	1.14	0.20	0.55	1.73

		λ = 1								
		$p = 1$			$p = 2$			$p = 3$		
n	m	$q = 1$	$q = 2$	$q = 3$	$q = 1$	$q = 2$	$q = 3$	$q = 1$	$q = 2$	$q = 3$
10	24	0.00	0.01	0.01	0.00	0.01	0.04	0.01	0.05	0.12
20	54	0.01	0.01	0.03	0.02	0.06	0.18	0.02	0.12	0.41
30	84	0.02	0.02	0.04	0.03	0.07	0.29	0.03	0.21	0.56
40	114	0.01	0.03	0.09	0.03	0.06	0.42	0.04	0.20	1.33
50	144	0.04	0.06	0.10	0.07	0.14	0.45	0.07	0.35	2.09
60	174	0.04	0.06	0.11	0.05	0.10	0.67	0.07	0.47	1.91
70	204	0.05	0.09	0.16	0.05	0.23	0.53	0.07	0.54	2.88
80	234	0.04	0.07	0.15	0.07	0.26	0.74	0.09	0.48	3.28
90	264	0.08	0.13	0.25	0.12	0.26	1.15	0.16	0.69	3.86
100	294	0.08	0.15	0.28	0.11	0.42	1.23	0.15	1.05	4.21

Table 2. Average computing time results for planar networks with n = 10 up to 100 nodes.

		$\lambda = 0$									$\lambda = 0.5$								
		$p = 1$			$p = 2$			$p = 3$			$p = 1$			$p = 2$			$p = 3$		
n	m	$q = 1$	$q = 2$	$q = 3$	$q = 1$	$q = 2$	$q = 3$	$q = 1$	$q = 2$	$q = 3$	$q = 1$	$q = 2$	$q = 3$	$q = 1$	$q = 2$	$q = 3$	$q = 1$	$q = 2$	$q = 3$
10	24	95.83	75.83	43.33	94.17	70.83	25.00	92.50	63.33	30.00	95.83	45.00	59.17	95.83	75.00	36.67	89.17	70.00	69.17
20	54	98.15	65.93	52.78	98.15	70.74	63.89	98.15	83.70	46.30	98.15	67.22	62.96	98.15	87.04	46.85	95.93	65.74	56.67
30	84	98.81	70.71	57.38	97.86	55.12	69.88	98.81	88.10	43.81	98.81	92.26	52.86	98.81	91.19	57.62	98.81	75.95	88.93
40	114	99.12	89.47	63.77	98.25	72.54	60.61	99.12	98.25	60.00	99.12	67.54	60.70	99.12	76.14	68.68	98.86	95.44	66.58
50	144	99.31	75.35	66.04	99.31	79.72	81.67	99.31	80.69	89.58	99.31	87.78	59.93	98.75	88.26	70.49	98.06	82.64	78.54
60	174	99.43	90.40	58.51	99.43	68.51	58.62	99.43	82.76	78.74	99.43	78.39	56.26	99.25	87.18	66.26	99.25	80.06	65.00
70	204	99.51	87.06	74.22	99.51	83.38	72.60	99.22	85.69	66.37	99.51	73.24	66.13	99.31	77.94	61.52	99.36	90.20	68.58
80	234	99.57	90.00	74.87	99.44	84.02	68.80	99.32	85.81	59.79	99.57	85.98	70.38	99.44	91.03	68.63	99.49	78.08	71.07
90	264	99.62	85.49	73.52	99.55	81.97	73.67	99.62	86.06	79.24	99.62	86.70	60.68	99.51	83.86	60.49	99.62	80.98	68.94
100	294	99.66	85.14	65.88	99.66	85.54	75.03	99.63	81.05	66.84	99.66	86.46	76.53	99.66	87.07	73.91	99.56	87.45	74.01

		$\lambda = 1$								
		$p = 1$			$p = 2$			$p = 3$		
n	m	$q = 1$	$q = 2$	$q = 3$	$q = 1$	$q = 2$	$q = 3$	$q = 1$	$q = 2$	$q = 3$
10	24	95.83	45.42	64.17	95.83	56.25	22.92	87.08	33.75	32.08
20	54	98.15	79.26	54.07	85.19	40.93	23.15	81.85	50.00	38.89
30	84	98.57	92.86	72.02	67.74	65.48	31.90	94.05	47.86	38.45
40	114	99.12	84.91	45.00	85.44	78.51	32.54	89.30	58.77	31.32
50	144	99.31	77.50	67.64	72.57	68.89	48.54	89.86	56.11	25.35
60	174	99.43	83.74	75.46	94.77	88.28	41.90	89.25	45.63	38.10
70	204	99.51	75.34	66.76	96.67	67.25	57.60	94.85	53.92	37.65
80	234	99.57	94.36	79.96	94.06	66.97	57.01	94.10	71.97	36.50
90	264	99.62	85.34	70.23	92.61	79.09	45.23	88.41	60.83	35.11
100	294	99.66	82.62	72.55	97.55	65.10	49.93	93.88	47.21	39.86

Table 3. Average percentage of edges removed for planar networks with $n = 10$ up to 100 nodes.

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